XXVII - Fragmente über die Grenzfälle der elliptischen Modulfunctionen - Originaux de Riemann

Auteurs : Bernhard Riemann

Les folios

En passant la souris sur une vignette, le titre de l'image apparaît.

16 Fichier(s)

Citer cette page

Bernhard Riemann, XXVII - Fragmente über die Grenzfälle der elliptischen Modulfunctionen - Originaux de Riemann, 1852

Emmylou Haffner (Institut des textes et manuscrits modernes, CNRS-ENS) ; Niedersächsische Staats- und Universitätsbibliothek, Göttingen ; projet EMAN (Thalim, CNRS-ENS-Sorbonne nouvelle).

Consulté le 01/10/2025 sur la plate-forme EMAN :
https://eman-archives.org/DWR/items/show/7

Droits

Présentation

Date1852
Auteur(s)Bernhard Riemann
Langue

- Allemand
- Latin

SourceCod. Ms. B. Riemann 4, p. 11-18

Format8 f., 16 p.
Nature du documentManuscrit original de Riemann

Description & Analyse

DescriptionOriginaux de Riemann
AnalysePas publié dans la traduction française.

Références lettres entre D et W.

Sujets

fonctions elliptiques, fonctions modulaires, Jacobi

Les relations du document

Collection ** Hors collections **

Ce document suivant :
XXVII - Fragmente über die Grenzfälle der elliptischen Modulfunctionen - Transcription de Dedekind

Afficher la visualisation des relations de la notice.

Notice créée par Richard Walter Notice créée le 18/10/2021 Dernière modification le 18/07/2022

Additamentum ad §um 40.

Formulae in hoc §o propositae, in eo casu, ubi modulus ipsius q unitatem aequat, consideratione satis dignae videntur, quippe quae functiones unius variabilis pro quovis ~~valore~~ argumenti valore discontinuas suppeditant praebeant.

~~Quin in hunc finem formulas 1–7 integremus~~

~~48) $\int \log k \frac{dq}{q}$~~ Series quidem propositae magna ex parte pro modulo ipsius q unitati aequali non convergunt, sed integrando series convergentes inde derivari possunt. Itaque primo integralia formularum 1–7 exhibemus ~~proponemus~~

$$48)\ \int_0 \left(\log k \frac{dq}{q} - \log 4\sqrt{q}\right)\frac{dq}{q} = -4\log(1+q) + \frac{4}{4}\log(1+q^2) - \frac{4}{9}\log(1+q^3) + \frac{4}{16}\log(1+ \ldots$$

$$49)\ \int_0 -\log k' \frac{dq}{q} = 4\log\frac{1+q}{1-q} + \frac{4}{9}\log\frac{1+q^3}{1-q^3} + \frac{4}{25}\log\frac{1+q^5}{1-q^5}$$

$$50)\ \int_0 \log\frac{2K}{\pi}\frac{dq}{q} = 4\log(1+q) + \frac{4}{9}\log(1+q^3) + \frac{4}{25}\log(1+q^5)$$

$$51)\ \int_0 \left(\frac{2K}{\pi}-1\right)\frac{dq}{q} = \cancel{4\log(1+q)} - 4\log(1-q) + \frac{4}{3}\log(1-q^3) - \frac{4}{5}\log(1-q^5)$$

$$= 2i\log\frac{1-qi}{1+qi} + \frac{2i}{2}\log\frac{1-q^2 i}{1+q^2 i} + \frac{2i}{3}\log\frac{1-q^3 i}{1+q^3 i}$$

$$52)\ \int_0 \frac{2kK}{\pi}\frac{dq}{q} = 4\log\frac{1+\sqrt{q}}{1-\sqrt{q}} - \frac{4}{3}\log\frac{1+\sqrt{q^3}}{1-\sqrt{q^3}} + \frac{4}{5}\log\frac{1+\sqrt{q^5}}{1-\sqrt{q^5}}$$

$$53)\ \int_0 \left(\frac{2kK}{\pi}-1\right)\frac{dq}{q} = 4i\log\frac{1-\sqrt{q}\,i}{1+\sqrt{q}\,i} + \frac{4i}{3}\log\frac{1-\sqrt{q^3}\,i}{1+\sqrt{q^3}\,i} + \frac{4i}{5}\log\frac{1-\sqrt{q^5}\,i}{1+\sqrt{q^5}\,i}$$

$$-\frac{4}{4}\log(1+q^2) + \frac{4}{3}\log(1+q^6) - \frac{4}{5}\log(1+q^{10})$$

$$= -\frac{2i}{4}\log\frac{1-q^2 i}{1+q^2 i} + \frac{2i}{12}\log\frac{1-q^6 i}{1+q^6 i} - \frac{2i}{20}\log\frac{1-q^{10} i}{1+q^{10} i}$$

54) $\int\left(\frac{2\sqrt{kk'}K}{\pi}-1\right)\frac{dq}{q} = -\frac{4i}{2}\log\left(1+q^{4}\right) + \frac{4}{8}\log\left(1+q^{8}\right) - \frac{4}{20}\log\left(1+q^{20}\right) + \frac{4}{14}\log\left(1+q^{14}\right)$

$$= -\frac{2i}{2}\log\frac{1-q^{2i}}{1+q^{2i}} + \frac{2i}{4}\log\frac{1-q^{4i}}{1+q^{4i}} - \frac{2i}{6}\log\frac{1-q^{6i}}{1+q^{6i}} + \frac{2i}{8}\log\frac{1-q^{8i}}{1+q^{8i}}$$

ubi logarithmos ita sumendos esse manifestum est, ut evanescant posito $q = 0$

Quantitas eadem ad dignitates ipsius q evoluta ~~hac serie~~ ~~expri~~ adhibitis Cl. Jacobi denotationibus hoc modo repraesentabitur

55) $\int_0 \left(\log k - \log 4\sqrt{q}\right)\frac{dq}{q} = -4\Sigma\frac{\varphi p}{p^2}\left(q^{p} - \frac{3q^{2p}}{4} - \frac{3}{16}q^{4p} - \frac{3}{64}q^{8p} - \frac{3}{256}q^{16p} - \ldots\right)$

56) $\int_0 -\log k'\,\frac{dq}{q} = 8\,\Sigma\,\frac{\varphi p}{p^2}\,q^{p}$

57) $\int_0 \log\frac{2K}{\pi}\,\frac{dq}{q} = 4\,\Sigma\,\frac{\varphi p}{p^2}\left(q^{p} - \frac{1}{2}q^{2p} - \frac{1}{4}q^{4p} - \frac{1}{8}q^{8p} - \frac{1}{16}q^{16p} - \ldots\right)$

58) $\int_0 \left(\frac{2K}{\pi}-1\right)\frac{dq}{q} = 4\,\Sigma\,\frac{\psi n\, q^{2(4m-1)^2 n}}{2(4m-1)^2 n}$

59) $\int_0 \left(\frac{2kK}{\pi}\right)\frac{dq}{q} = 8\,\Sigma\,\frac{\psi(n)\, q^{\frac{(4m-1)^2 n}{2}}}{(4m-1)^2 n}$

60) $\int_0 \left(\frac{2k'^{m}K}{\pi}-1\right)\frac{dq}{q} = -4\,\Sigma\,\frac{\psi(n)\, q^{(4m-1)^2 n}}{(4m-1)^2 n} + 4\,\Sigma\,\frac{\psi(n)\, q^{2^{l+1}(4m-1)^2 n}}{2^{l+1}(4m-1)^2 n}$

61) $\int_0 \left(\frac{2\sqrt{k'}\,K}{\pi}-1\right)\frac{dq}{q} = -4\,\Sigma\,\frac{\psi(n)\, q^{2(4m-1)^2 n}}{2(4m-1)^2 n} + 4\,\Sigma\,\frac{\psi n\, q^{2^{l+2}(4m-1)^2 n}}{2^{l+2}(4m-1)^2 n}$

Jam (demonstremus) ~~lemma~~ ~~generale~~ theorema sequens generale

Si summa seriei ~~Σa_n~~ ~~convergit~~

$= a_0 + a_1 + a_2$

ea, quae quoque ordine summata summam habet convergentem,

Accuratiori ~~huget~~ similem propositionem disquisitionem [illegible] ~~eam~~ ~~calculandam~~, ~~et~~ [illegible] ~~alla~~ [illegible]

[illegible] theorema sequens generale

functio cujus r huic seriei per seriem

definita $a_0 + a_1 r + a_2 r^2 \ldots$

est potestas, convergente r ~~[illegible]~~ versus limitem 1, convergit versus valorem

eundem.

Unde sequitur facile deducitur, minute modulo ejusdem q infra unitatem ~~[illegible]~~ modulis [illegible] ~~[illegible]~~ [illegible]

si functio $f(q)$ complexae quantitatis q exhibeatur per seriem,

$a_0 + a_1 q + a_2 q^2 \ldots$ cujus modulus et unitas

convergentem [illegible] quantitate aequale, si habeat summam pro

[illegible] [illegible] modulo [illegible] quantitate [illegible] datur ~~[illegible]~~ convergente q versus q_0

primum valorem eum, [illegible] functio $f(q)$ aequalis

ita, ut modulus tantum [illegible] geometricam

hic [illegible] [illegible] puncto, per quod quantitas q repraesentatur, ~~[illegible]~~ in linea ad limitem quem [illegible] functio est [illegible]

normali

Quamdiu [illegible] in serie hos tantum valores functionum propositarum hic respicimus, [illegible] evolutionis ~~48 – 54~~ latius patet

[illegible]

~~[illegible]~~ Sit brevitatis gratia (x) quantitas absolute minima quantitatum, a quantitate x numero integro distantium, aut, si

x numero integro et fractione $\frac{1}{2}$ composita est, $= 0$; porro

$E(x)$ numerus integer maximus non major quam x: obtinemus e

~~[illegible]~~ attribuendo ipsi q valorem $q_0 = e^{xi}$

48, ~~[illegible]~~

$$62)\ \int_0^{x} \left(\log \frac{dq}{q} - \log 4 \sqrt[4]{q}\right) \frac{dq}{q} = -2 \log 4 \cos \frac{x}{2}^2 + \frac{2}{4} \log 4 \cos \frac{2x}{2}^2 - \frac{2}{9} \log 4 \cos \frac{3x}{2} + \frac{2}{16} \log \ldots$$

$$- \frac{2\pi i}{1} \cdot \left(\frac{x}{2\pi}\right) + \frac{4\pi i}{4} \left(\frac{2x}{2\pi}\right) - \frac{4\pi i}{9} \left(\frac{3x}{2\pi}\right) + \frac{4\pi i}{16} \left(\frac{4x}{2\pi}\right)$$

$$= \frac{\Sigma (-1)^n \log 4 \cos \frac{nx}{2}^2}{nn} - 2 \log 4 \left(1 - \frac{1}{4} + \frac{1}{9} - \frac{1}{16}\right)$$

· Pars imaginaria, hujus seriei convergit, quicunque est valor ipsius x, pars realis si designantibus m, n numeros integros inter se primos, et posito $\frac{x}{2\pi} = \frac{m}{n}$, ita exhiberi potest:

1° si n est impar, aequalis fit,

$$\frac{\pi^2}{n^2}\sum_{1,n-1}^{s}(-1)^s \frac{\cos\frac{\pi s}{n}}{\sin\frac{\pi s}{n}^2}\log 4\cos\frac{s m\pi}{n}^2 - \frac{\pi^2}{6^2 n^2}\log q$$

2° si n est par, designante p numerum ~~primum~~ imparem

$$= \frac{\pi^2}{n^2}\sum_{1}^{\frac{n}{2}-1} \frac{2(-1)^s\log 4\cos\frac{s m\pi}{n}^2}{\sin\pi\frac{s}{n}^2} + \frac{\pi^2}{3n^2}\log q + \frac{n\pi^2}{n^2}(-1)^{\frac{m}{2}}\left(\log\frac{q-q}{q+q} + \log[illegible] + \frac{8}{\pi^2}\Sigma\frac{\log p}{p^2}\right)$$

manifesto ~~[illegible]~~, ita ut intelligitur quae formula ~~[illegible]~~ ~~convergere~~ functionem propositam ~~[illegible]~~ proposuisse ~~[illegible]~~ = alterata functione $\frac{2\pi^2}{n^2}(-1)^{\frac{m}{2}}\log\frac{q-q}{q+q}$, si q convergat [illegible] versus limitem q_1, convergere versus limitem finitum qui que valorem assignat.

Porro inde obtinetur

$$\int_0^{x} \log x\frac{dq}{q} = 2\log\operatorname{tg}\frac{x}{2}^2 - \frac{2}{q}\log\operatorname{tg}\frac{8x}{2}^2 - \frac{2}{8x}\log\operatorname{tg}\frac{8x}{2}^2$$

- si x est (quantitas) numerus [illegible], non convergit, [illegible]. Designantibus m, n numeros integros inter se primos, et posito $\frac{x}{2\pi} = \frac{m}{n}$ ita exhiberi potest

$$+4\pi i\left(\left(\frac{x}{2\pi}\right)-\left(\frac{x}{2\pi}+\frac{1}{2}\right)\right)+\frac{4\pi i}{9}\left(\left(\frac{3x}{2\pi}\right)-\left(\frac{3x}{2\pi}+\frac{1}{2}\right)\right)+\frac{4\pi i}{25}\left(\left(\frac{5x}{\pi}\right)-\left(\frac{5x}{2\pi}+\frac{1}{2}\right)\right)$$

$$(3)\int_0^{e^{xi}} \log^2 \frac{\partial q}{q} = -2\log \operatorname{tg}\frac{x}{2}^2-\frac{2}{9}\log \operatorname{tg}\left(\frac{3x}{2}\right)^2-\frac{2}{25}\log \operatorname{tg}\left(\frac{5x}{2}\right)^2$$

$$\frac{2\pi}{2\pi}\frac{\partial q}{q} = -\sum_{-\infty,\infty}\frac{\log \operatorname{tg}\left(\frac{px}{2}\right)^2}{p^2}$$

$$(4)\int_0^{e^{xi}} \log\frac{2\mathfrak{X}}{\pi}\frac{\partial q}{q} = -2\log 4\cos\left(\frac{x}{2}\right)^2+\frac{2}{9}\log 4\cos\frac{3x}{2}^2 \quad = \sum_{\infty}\frac{\log 4\cos\frac{px}{2}^2}{p^2}$$

$$+4\pi i\left(\frac{x}{2\pi}\right)+\frac{4\pi i}{9}\left(\frac{3x}{2\pi}\right)+\frac{4\pi i}{25}\left(\frac{5x}{2\pi}\right)$$

$$(5)\int_0^{e^{xi}}\left(\frac{2\mathfrak{X}}{\pi}-1\right)\frac{\partial q}{q} = -2\log 4\sin\frac{x}{2}^2+\frac{2}{3}\log 4\sin\frac{3x}{2}^2-\frac{2}{5}\log 4\sin\left(\frac{5x}{2}\right)^2$$

$$-4\pi i\left(\frac{x}{2\pi}+\frac{1}{2}\right)+\frac{4\pi i}{3}\left(\frac{3x}{2\pi}+\frac{1}{2}\right)$$

$$(6)\ \cancel{\int^{xi}\frac{2\mathfrak{X}\partial q}{\pi}} = i\log \operatorname{tg}\left(\frac{x+\pi}{4}\right)^2-\frac{i}{3}\log \operatorname{tg}\left(\frac{3x+\pi}{4}\right)^2+\frac{i}{5}\log \operatorname{tg}\left(\frac{5x+\pi}{4}\right)^2$$

$$2\pi\left(\left(\frac{x}{2\pi}+\frac{1}{4}\right)-\left(\frac{x}{2\pi}+\frac{3}{4}\right)\right)+\frac{2\pi}{3}\left(\left(\frac{3x}{2\pi}+\frac{1}{4}\right)-\left(\frac{3x}{2\pi}+\frac{3}{4}\right)\right)+\frac{2\pi}{5}\left(\left(\frac{5x}{2\pi}+\frac{1}{4}\right)-\left(\frac{5x}{2\pi}+\frac{3}{4}\right)\right)$$

$$(6)\int_0^{e^{xi}}\frac{2\mathfrak{X}}{\pi}\frac{\partial q}{q} = -2\log \operatorname{tg}\frac{x}{4}^2+\frac{2}{3}\log \operatorname{tg}\frac{3x}{4}^2-\frac{2}{5}\log \operatorname{tg}\frac{5x}{4}^2$$

$$-4\pi i\left(\left(\frac{x}{4\pi}\right)-\left(\frac{x}{2\pi}+\frac{1}{2}\right)\right)-\frac{4\pi i}{3}\left(\frac{3x}{2\pi}-\frac{3x}{2\pi}+\frac{1}{2}\right)$$

$$(6^*)\int_0^{e^{xi}} = i\log \operatorname{tg}\left(\frac{x+\pi}{4}\right)^2-\frac{i}{3}\log \operatorname{tg}\frac{3x+\pi}{4}^2+\frac{i}{5}\log \operatorname{tg}\frac{5x+\pi}{4}^2$$

$$+4\pi\left(\frac{x}{4\pi}+\frac{1}{4}-\left(\frac{x}{4\pi}+\frac{3}{4}\right)\right)+\frac{4\pi}{3}\left(\left(\frac{3x}{4\pi}+\frac{1}{4}\right)-\left(\frac{3x}{4\pi}+\frac{3}{4}\right)\right)$$

$$(7)\int_0^{e^{xi}}\left(\frac{2\mathfrak{X}}{\pi}-1\right)\frac{\partial q}{q} = -2\log 4\cos\frac{x}{2}^2+\frac{2}{3}\log 4\cos\frac{3x}{2}^2-\frac{2}{5}\log 4\cos\frac{5x}{2}^2$$

$$-4\pi i\left(\frac{x}{2\pi}\right)+\frac{4\pi i}{9}\left(\frac{3x}{2\pi}\right)-\frac{4\pi i}{25}\left(\frac{5x}{2\pi}\right)$$

$$= -i\log \operatorname{tg}\left(\frac{x+\pi}{4}\right)^2+\frac{i}{3}\log \operatorname{tg}\left(\frac{3x+\pi}{4}\right)^2-\frac{i}{5}\log \operatorname{tg}\frac{5x+\pi}{4}$$

$$-2\pi\left(\left(\frac{x}{2\pi}+\frac{1}{4}\right)-\left(\frac{x}{2\pi}+\frac{3}{4}\right)\right)+\frac{2\pi}{3}\left(\left(\frac{3x}{2\pi}+\frac{1}{4}\right)-\left(\frac{3x}{2\pi}+\frac{3}{4}\right)\right)$$

(8) $\left(2\frac{\sqrt{\pi^2 x}}{\pi}-1\right)\frac{\pi i}{4} = -\log 4\cos^2 x^2 + \frac{1}{3}\log 4\cos^2 3x^2 - \frac{1}{5}\log \cos 5x^2$

$$-2\pi i\left(\frac{x}{\pi}\right) + \frac{2\pi i}{3}\left(\frac{3x}{\pi}\right) - \frac{2\pi i}{5}\left(\frac{5x}{\pi}\right)$$

$$= -\frac{i}{2}\log \operatorname{tg}\left(x+\frac{\pi}{4}\right)^2 + \frac{i}{4}\log \operatorname{tg}\left(2x+\frac{\pi}{4}\right)^2 - \frac{i}{6}\log \operatorname{tg}\left(3x+\frac{\pi}{4}\right)^2$$

$$-\pi\left(\left(\frac{x}{\pi}+\frac{1}{4}\right)-\left(\frac{x}{\pi}+\frac{3}{4}\right)\right)+\frac{\pi}{2}\left(\left(\frac{2x}{\pi}+\frac{1}{4}\right)-\left(\frac{2x}{\pi}+\frac{3}{4}\right)\right)-\frac{\pi}{3}\left(\left(\frac{3x}{\pi}+\frac{1}{4}\right)-\left(\frac{3x}{\pi}+\frac{3}{4}\right)\right)$$

$$(x) = \sum_{1,\infty} \frac{(nx)}{n^2}$$

$$\varphi x = \frac{(x)}{1} + \frac{(2x)}{4} + \frac{(3x)}{9} +$$

$$x = \frac{m}{2n}$$

$$= \sum \frac{((2s+1)x)}{n^2(2s+1)^2}$$

$$= \frac{1}{n^2}\,\frac{1}{2} \sum \frac{1}{(2s+1)^2}$$

$$= \frac{\pi^2}{16 n^2}$$

$$\Sigma (f x) x_1$$

$$2\left(\frac{\pi^2}{16} + 2\frac{\pi^2}{16 \cdot 9} + 4\frac{\pi^2}{16 \cdot 16} + 4\frac{\pi^2}{16}\right.$$

Posito $x = \frac{m}{n} 2\pi$ fit pars imaginaria formulae 65 15

1° si n est numerus par

$$\overline{\sum_{0,\infty}^{s}} - 4\pi i \sum_{1,2n-1}^{p} (-1)^{\frac{p-1}{2}} \frac{1}{p+2ns} \left(\frac{pm}{n} + \frac{1}{2}\right) (-1)^{\frac{m}{2}s}$$

2° si n est numerus impar

$$= \sum_{0,\infty}^{s} - 4\pi i \sum_{1,2n-1}^{p} (-1)^{\frac{p-1}{2}} \frac{1}{p+2ns} \left(\frac{pm}{n} + \frac{1}{2}\right) (-1)^{s}$$

quam potest habere si valorem finitum, nisi n est $\equiv 0 \bmod 4$.

~~[illegible]~~

~~[illegible]~~ (n, m)

$$f_r = a_0 + a_1 r + a_2 r^2 \cdots a_r r^n + (\mathcal{L}_{r+1} - \mathcal{L}_r) r^{n+1} + (\mathcal{L}_{r+2} - \mathcal{L}_{r+1}) r^{n+2} \cdots$$

$$= a_0 + a_1 r + a_2 r^2 \cdots a_r r^n + \ldots (r^{n+1} - r^{n+2}) + (\mathcal{L}_{r+2} - \mathcal{L}_{r+1})$$

$$f_r = a_0 + a_1 r \cdots a_r r^n + \varepsilon_{n+1} r^{n+1} + (\varepsilon_{n+2} - \varepsilon_{n+1}) r^{n+2} + (\varepsilon_{n+3} - \varepsilon_{n+2}) r^{n+3}$$

$$= a_0 + a_1 r \qquad a_n r^n - \varepsilon_{n+1}(r^{n+1} - r^{n+2}) + \varepsilon_{n+2}(r^{n+2} - r^{n+3}) + \varepsilon_{n+3}(r^{n+3} - r^{n+4})$$

Convergentia summae

$$a_0 + a_1 + a_2 + a_3 \cdots$$

postulat, ut ~~assignata~~ ~~[illegible]~~ data quantitate quamvis parva ~~termini~~ a_n ~~quomodo~~, ~~ut~~

ε ~~[illegible]~~ assignari posset ~~numerus~~ ~~[illegible]~~

~~[illegible] proportio usque ad terminum quemvis [illegible]~~ summa

a qu ~~summa~~ $a_n + a_{r+1}$ summa usque ad terminum quemvis

~~[illegible]~~ ~~[illegible]~~ ~~[illegible]~~ Tam ~~[illegible]~~ posito ~~assignata~~ bene

[illegible] incertitudes valorum absolutam ipso ε minorem tatis gratia

$\varepsilon_{n+1} = a_{n+1}$

$\varepsilon_{n+2} = a_{n+1} + a_{n+2}$

$\varepsilon_{n+3} = a_{n+1} + a_{n+2} + a_{n+3}$

functio $f(x) = a_0 + a_1 x + a_2 x^2 \ldots$ ~~quae demum~~ ~~quum $n+1 = f(x)$~~ denotetur,

~~transformatur facile~~ sub hac forma exhibetur

$= a_0 + a_1 + a_2 x \ldots a_n x^n + \varepsilon_{n+1} x^{n+1} + (\varepsilon_{n+2} - \varepsilon_{n+1}) x^{n+2} + (\varepsilon_{n+3} - \varepsilon_{n+2}) x^{n+3}$

$= a_0 \ldots a_n x^n + \varepsilon_{n+1}(x^{n+1} - x^{n+2}) + \varepsilon_{n+2}(x^{n+2} - x^{n+3}) + \varepsilon_{n+3}$

Unde patet ~~approprinquare~~ convergentiae versus limitem functionis valorem pro

fit tandem quavis quantitate data minus ... destare

valor $a_1 + a_2$

summa terminorum altioris gradus quam n, quum sint ... infinito.

ex hyp. omnes omnino signo $< \varepsilon$ differentiae $x^{n+1} - x^{n+2}$

positivae, manifesto evadit quantitate absoluta

$< \varepsilon(x^{n+1} - x^{n+2}) + \varepsilon(x^{n+2} - x^{n+3}) \ldots$

$< \varepsilon\, x^{n+1}$ non altioris gradus quam n, at

Summa autem terminorum ~~inferiorum~~ approximandorum constat ~~evidenter~~

functio algebraica ipsius x, quam

... ipso ~~data~~ determinato ...

~~valor~~ $a_1 + a_2 \ldots a_n$

... approximatione posse ... potest approprinquare differentiam functionis post n valore seriei

$a_0 + a_1 \ldots\ldots$

infra quantitatem quamvis datam descendere

Ex hoc theoremate, quod Cl. Abel tribuendum esse a Cl. Dirichlet ~~audivi~~ (1852 Sept. 14.) ~~accepi quem~~ ~~quem hoc scrip.~~ ~~quam antea~~ ~~mihi~~ jam ~~scripsi~~ ~~nominis~~ ~~monuit~~ ~~monuit~~ ~~supra~~ in antecedentibus jam scriptis ; facile deducitur.

~~majorem~~

nec tati

v. unitati

(3) x impar $\quad = -\sum_{1,2x-1}^{p} \sum_{-\infty,\infty}^{t} \frac{\log \operatorname{tg} \frac{px}{2x}^2}{(2xt+p)} \qquad \frac{\lambda}{2\pi} = \frac{m}{n}$

x par
$$= -\sum_{-\infty,\infty}^{t} \sum_{1,x-1}^{p} \frac{\log \operatorname{tg} \frac{px}{2}^2}{(xt+p)^2}$$

(64) x impar $\quad = \sum_{1,2x-1}^{p} \sum_{-\infty,\infty}^{t} \frac{\log 4 \cos \frac{px}{2}^2}{(2xt+p)^2}$

x pair
$$= + \sum_{1,x-1}^{p} \sum_{-\infty,\infty}^{t} \frac{\log 4 \cos \left(\frac{px}{2}\right)^2}{(xt+p)^2}$$

62) x impair