Trois modules (ou groupes abéliens)

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En passant la souris sur une vignette, le titre de l'image apparaît.

2 Fichier(s)

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Présentation

TitreTrois modules (ou groupes abéliens)
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Sujet

- groupes
- Modulgesetz
- trois modules

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Description & Analyse

DescriptionCommence par une étude du "groupe" généré par 3 modules ou trois groupes abéliens. Reformulation dans la notation utilisée pour la théorie des groupes (eg Modulgesetz). Étude du treillis formé par les sous-groupes normaux.
NotesRédigé par dessus une lettre.
Mode(s) d'écriture

- Calculs
- Tableau

Auteur·es de la descriptionHaffner, Emmylou

Mots-clefs

Groupes, Modulgesetz, trois modules
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Drei Moduln (oder Abelsche Gruppen) a, b, c mit der Annahme $b < c$, a, b

$$a''' = b + c = b \;;\; a_3 = b - c = c$$

$a''' = b$; $b''' = c + a$; $c''' = a + b$; $a_3 = c$; $b_3 = c - a$; $c_3 = a - b$

$\mathfrak{d}'''' = c'''$, $\mathfrak{d}_4 = b_3$, $\mathfrak{d}' = b - b'''$; $\mathfrak{d}_1 = c + c_3$

$a'' = b'''$, $b'' = b$, $c'' = b - b'''$; $a_2 = c_3$, $b_2 = c + c_3$, $c_2 = c$

$a' = b'''$, $b' = b$, $c' = c + c_3$; $a_1 = c_3$, $b_1 = b - b'''$, $c_1 = c$

$a_0 = c + c_3$, $b_0 = b_2 - b'''$, $c_0 = c + c_3$

$= b - b'''$

$(c_3, b) = 1 = h c b$; $h = 1$, $f = 1$, $b_1 = 1$

$a'' = a'$	$a_2 = a_1$	$\mathfrak{d}''' = c'''$, $a'' = b''$, $b''' = a''$, $c'' = \mathfrak{d}'$, $c' = c_0$, $c = c_1$
$b'' = b'$	$b_2 = b_1$	~~[illegible]~~
$c'' = c'$	$c_2 = c_1$	$b_3 = \mathfrak{d}_4$; $a_2 = c_3$, $c_2 = a_3$, $\mathfrak{d}_1 = b_2$, $b_0 = b_1$, $b' = b$

$\mathfrak{d}' = a_0 = b_0 = c_0 = \mathfrak{d}_1$

Also

$\mathfrak{d}' = a_0 = b_0 = c_0 = \mathfrak{d}_1 = c'' = c' = b_2 = b_1$; $\mathfrak{d}'''' = c'''$; $\mathfrak{d}_4 = b_3$

$a'' = a' = b'''$; $a_2 = a_1 = c_3$

$b'' = b' = b$; $c_2 = c_1 = c = a_3$

$= a'''$

Also bleiben 8 Moduln

c''' ; b, a' ; a_0 ; a, c ; a_1 ; c_3

	X' c'''	X b	Ψ' a'	H a	$X\Psi'$ a_0	Ψ c	HX a_1	$H\Psi$ b_3
X'		X	Ψ'	H	$X\Psi'$	Ψ	HX	$H\Psi$
X	X'		$X\Psi'$	HX	$X\Psi'$	Ψ	HX	$H\Psi$
Ψ'	X'	X'		H	$X\Psi'$	Ψ	HX	$H\Psi$
H	X'	X'	Ψ'		HX	$H\Psi$	HX	$H\Psi$
$X\Psi'$	X'	X	Ψ'	Ψ'		Ψ	HX	$H\Psi$
Ψ	X'	X	Ψ'	Ψ'	$X\Psi'$		$H\Psi$	$H\Psi$
HX	X'	X	Ψ'	H	$X\Psi'$	$X\Psi'$		$H\Psi$
$H\Psi$	X'	X	Ψ'	H	$X\Psi'$	Ψ	XH	

$\underbrace{\qquad\qquad}_{\text{u. s. w.}}$

$$\frac{c + (a - b)}{= (c + a) - b}$$

$c''' < a', b$

$a' < a, a_0$

$b < a_0$

$a < a_1$

$a_0 < a_1, c$

$a_1 < b_3$

$c < b_3$

$X' < X, \Psi'$

$X < X\Psi'$

$\Psi' < X\Psi'$, H

$X\Psi' < HX, \Psi$

$H < HX$

$HX < H\Psi$

$\Psi < H\Psi$

NB. Theiler von XH und $X\Psi' = \Psi'$

$= X$ [Theiler von H und $X\Psi' = \Psi'$],

$= X\Psi'$

$1 < X < \Psi < \Phi$ und X Normaltheiler von Ψ

$1 < H < \Phi$ und H Normaltheiler von Φ.

$\Psi' = H + \Psi$; ~~[illegible]~~ Normaltheiler von Ψ

$X' = H + X = X + \Psi'$ Normaltheiler von Ψ

~~[illegible]~~

$H - \Psi = H\Psi$

$H - X = HX$

$X - \Psi' = X\Psi' = HX + \Psi$; in der That gilt hier das Modulgesetz $X - (H + \Psi) = (X - H) + \Psi$

$c'' = (\Psi, H) = (\Psi', H) = (\Psi, H\Psi)$	$e' = (H\Psi, \Phi)$
$f' = (X', \Psi') = (X, \Psi') = (X, X\Psi')$	$f' = (X\Psi', \Psi) = (HX + \Psi, \Psi) = (HX, \Psi) = (HX, H\Psi)$
$g'' = (1, X')$	$g' = (X', X) = (\Psi', X) = (\Psi', X\Psi') = (H, X) = (H, HX)$

$\boxed{\Phi = \mathfrak{M}\,;\ X = \mathfrak{M}'\,;\ \Psi = \mathfrak{P}}$

m natürliche Zahl

$\mathfrak{M}$ Gruppe der $\varphi(m)$ Classen (mod. m), welche relative Primzahlen zu m enthalten

$\mathfrak{H}$ eine in $\mathfrak{M}$ enthaltene Gruppe (Theiler von $\mathfrak{M}$)

$(\mathfrak{H}, \mathfrak{M}) = n$ Index von $\mathfrak{H}$ nach $\mathfrak{M}$; $\varphi(m) = nn'$, $n' = (1, \mathfrak{H})$

p eine natürliche Primzahl

p' die höchste in m aufgehende Potenz von p

$m = m'p'$; m' der grösste durch p nicht theilbare Divisor von m.

$\mathfrak{M}'$ ~~derjenige Theiler von~~ Inbegriff derjenigen Classen (mod. m) in $\mathfrak{M}$, deren Zahlen $\equiv 1$ (mod. m')

$\mathfrak{P}$ Inbegriff derjenigen Classen (mod. m) in $\mathfrak{M}$, deren Zahlen $\equiv 1, p, p^2, p^3 \ldots$ (mod. m')

$(1, \mathfrak{M}') = \varphi(p')$; $(\mathfrak{M}', \mathfrak{M}) = \varphi(m')$ | Erst $\mathfrak{P}$, dann $\mathfrak{M}'$ definiren ; $\mathfrak{M}'$ Theiler von $\mathfrak{P}$ |

$e_0 = (\mathfrak{P}, \mathfrak{M})$, $f_0 = (\mathfrak{M}', \mathfrak{P})$, $g_0 = (1, \mathfrak{M}') = \varphi(p')$; $e_0 f_0 = \varphi(m')$

$\left.\begin{array}{l} \mathfrak{A}+\mathfrak{B} \text{ gr. gem. Theiler der Gruppen } \mathfrak{A}, \mathfrak{B} \\ \mathfrak{A}-\mathfrak{B} = \mathfrak{A}\mathfrak{B} \text{ kl. gem. Vielfache der Gruppen } \mathfrak{A}, \mathfrak{B} \end{array}\right\}$ $(\mathfrak{A}, \mathfrak{B}) = (\mathfrak{A}+\mathfrak{B}, \mathfrak{B}) = (\mathfrak{A}, \mathfrak{A}-\mathfrak{B})$

$e = (\mathfrak{H}\mathfrak{P}, \mathfrak{M})$, $f = (\mathfrak{H}\mathfrak{M}', \mathfrak{H}\mathfrak{P})$, $g = (\mathfrak{H}, \mathfrak{H}\mathfrak{M}')$; $efg = n$

$f = (\mathfrak{H}-\mathfrak{M}', \mathfrak{H}-\mathfrak{P}) = (\mathfrak{H}-\mathfrak{M}', \mathfrak{P}) = (\mathfrak{P}+(\mathfrak{H}-\mathfrak{M}'), \mathfrak{P}) = ((\mathfrak{P}+\mathfrak{H})-\mathfrak{M}', \mathfrak{P})$

$g = (\mathfrak{H}, \mathfrak{H}-\mathfrak{M}') = (\mathfrak{H}, \mathfrak{M}') = (\mathfrak{H}+\mathfrak{M}', \mathfrak{M}')$

$1 < \mathfrak{M}' < \mathfrak{P} < \mathfrak{M}$; $1 < \mathfrak{H} < \mathfrak{M}$

$1 < \mathfrak{H}+\mathfrak{M}' < \mathfrak{H}+\mathfrak{P} < \mathfrak{H} < \mathfrak{H}-\mathfrak{M}' < \mathfrak{H}-\mathfrak{P} < \mathfrak{M}$

$(\mathfrak{H}, \mathfrak{M}') = (\mathfrak{H}+\mathfrak{M}', \mathfrak{M}') = (\mathfrak{H}, \mathfrak{H}-\mathfrak{M}')$; $(\mathfrak{H}, \mathfrak{P}) = (\mathfrak{H}-\mathfrak{P}, \mathfrak{P}) = (\mathfrak{H}, \mathfrak{H}-\mathfrak{P})$

$(\mathfrak{M}', \mathfrak{H}) = (\mathfrak{H}+\mathfrak{M}', \mathfrak{H}) = (\mathfrak{M}', \mathfrak{H}-\mathfrak{M}')$; $(\mathfrak{P}, \mathfrak{H}) = (\mathfrak{H}+\mathfrak{P}, \mathfrak{H}) = (\mathfrak{P}, \mathfrak{H}-\mathfrak{P})$

$\mathfrak{H}+\mathfrak{M}' = \mathfrak{H}''$, $\mathfrak{H}-\mathfrak{M}' = \mathfrak{H}_1$ | $\mathfrak{H}_0 = \mathfrak{P}+(\mathfrak{H}-\mathfrak{M}') = (\mathfrak{H}+\mathfrak{P})-\mathfrak{M}'$

$\mathfrak{H}+\mathfrak{P} = \mathfrak{H}'$; $\mathfrak{H}-\mathfrak{P} = \mathfrak{H}_2$ | $\mathfrak{H}_0 = \mathfrak{P}+\mathfrak{H}_1 = \mathfrak{H}'-\mathfrak{M}'$

$1 < \mathfrak{H}''$; $\mathfrak{H}'' < \mathfrak{H}', \mathfrak{M}'$; $\mathfrak{H}' < \mathfrak{H}, \mathfrak{H}_0$; $\mathfrak{M}' < \mathfrak{H}_0$; $\mathfrak{H} < \mathfrak{H}_1$; $\mathfrak{H}_0 < \mathfrak{H}_1, \mathfrak{P}$; $\mathfrak{H}_1 < \mathfrak{H}_2$; $\mathfrak{P} < \mathfrak{H}_2$; $\mathfrak{H}_2 < \mathfrak{M}$

$\left\{\begin{array}{l} f = (\mathfrak{H}_0, \mathfrak{P}) = (\mathfrak{H}_1, \mathfrak{H}_2) = (\mathfrak{H}', \mathfrak{P}) \\ g = (\mathfrak{H}, \mathfrak{H}_1) = (\mathfrak{H}'', \mathfrak{M}') = (\mathfrak{H}, \mathfrak{M}') \\ e = (\mathfrak{H}_2, \mathfrak{M}) \end{array}\right\}$

$\left\{\begin{array}{l} e_0 = (\mathfrak{P}, \mathfrak{M}) = (\mathfrak{P}, \mathfrak{H}_2)(\mathfrak{H}_2, \mathfrak{M}) = e\,(\mathfrak{P}, \mathfrak{H}_2) \\ f_0 = (\mathfrak{M}', \mathfrak{P}) = (\mathfrak{M}', \mathfrak{H}_0)(\mathfrak{H}_0, \mathfrak{P}) = f\,(\mathfrak{M}', \mathfrak{H}_0) \\ g_0 = (1, \mathfrak{M}') = (1, \mathfrak{H}'')(\mathfrak{H}'', \mathfrak{M}') = g\,(1, \mathfrak{H}'') \end{array}\right\}$

$\left\{\begin{array}{l} e' = (\mathfrak{P}, \mathfrak{H}_2) = (\mathfrak{P}, \mathfrak{H}-\mathfrak{P}) = (\mathfrak{P}, \mathfrak{H}) = (\mathfrak{H}', \mathfrak{H}) \\ f' = (\mathfrak{M}', \mathfrak{H}_0) = (\mathfrak{M}', \mathfrak{H}'-\mathfrak{M}') = (\mathfrak{M}', \mathfrak{H}') = (\mathfrak{H}'', \mathfrak{H}') \\ g' = (1, \mathfrak{H}'') \end{array}\right\}$ $\begin{array}{l} e_0 = ee' \\ f_0 = ff' \\ g_0 = gg' \end{array}$ $\begin{array}{|l} e_0 f_0 g_0 = \varphi(m) \\ efg = n \\ e'f'g' = n' \end{array}$

$g_0 = \varphi(p')$; $e_0 f_0 = \varphi(m')$

$p = (y_1 y_2 \cdots y_e)^g$; $N(y_r) = p^f$

$y_r = (y_{r,1}\, y_{r,2} \cdots y_{r,e'})^{g'}$; $\pi(y_{r,s}) = y_r^{f'}$; $\pi(y_r) = y_r^{e'f'g'} = y_r^{n'}$

$p = \prod^{ee'} y_{r,s}^{gg'} = (\prod^{ee'} y_{r,s})^g$ $\quad N_0(y_{r,s}) = N\pi(y_{r,s}) = N(y_r^{f'}) = p^{ff'} = p^{f_0}$