Trois modules (ou groupes abéliens)

Auteurs : Dedekind, Richard

En passant la souris sur une vignette, le titre de l'image apparaît.

2 Fichier(s)

Contributeur·ricesHaffner, Emmylou
ÉditeursEmmylou Haffner (Institut des textes et manuscrits modernes, CNRS-ENS) ; Niedersächsische Staats- und Universitätsbibliothek, Göttingen ; projet EMAN (Thalim, CNRS-ENS-Sorbonne nouvelle).

Présentation

TitreTrois modules (ou groupes abéliens)
Date 189x
Sujet

- groupes
- Modulgesetz
- trois modules

CoteCod. Ms. Dedekind X 11-2, p. 14
Format1 p., 2 f.
LangueAllemand

Description & Analyse

DescriptionCommence par une étude du "groupe" généré par 3 modules ou trois groupes abéliens. Reformulation dans la notation utilisée pour la théorie des groupes (eg Modulgesetz). Étude du treillis formé par les sous-groupes normaux.
NotesRédigé par dessus une lettre.
Mode(s) d'écriture

- Calculs
- Tableau

Auteur·es de la descriptionHaffner, Emmylou

Mots-clefs

Groupes, Modulgesetz, trois modules
Notice créée par Emmylou Haffner Notice créée le 19/07/2021 Dernière modification le 21/07/2021

Drei Moduln (oder Abel'sche Gruppen) $\mathfrak{a}, \mathfrak{b}, \mathfrak{c}$ mit der Annahme $\mathfrak{b} < \mathfrak{c}$, $\mathfrak{a}, \mathfrak{b}$

$$\mathfrak{a}''' = \mathfrak{b}+\mathfrak{c} = \mathfrak{b}\,;\ \mathfrak{a}_3 = \mathfrak{b}-\mathfrak{c} = \mathfrak{c}$$

$\mathfrak{a}''' = \mathfrak{b}$, $\mathfrak{b}''' = \mathfrak{c}+\mathfrak{a}$, $\mathfrak{c}''' = \mathfrak{a}+\mathfrak{b}$; $\mathfrak{a}_3 = \mathfrak{c}$, $\mathfrak{b}_3 = \mathfrak{c}-\mathfrak{a}$; $\mathfrak{c}_3 = \mathfrak{a}-\mathfrak{b}$

$\mathfrak{d}''' = \mathfrak{c}'''$, $\mathfrak{d}_4 = \mathfrak{b}_3$, $\mathfrak{d}' = \mathfrak{b}-\mathfrak{b}'''$; $\mathfrak{d}_1 = \mathfrak{c}+\mathfrak{c}_3$

$\mathfrak{a}'' = \mathfrak{b}'''$, $\mathfrak{b}'' = \mathfrak{b}$, $\mathfrak{c}'' = \mathfrak{b}-\mathfrak{b}'''$; $\mathfrak{a}_2 = \mathfrak{c}_3$, $\mathfrak{b}_2 = \mathfrak{c}+\mathfrak{c}_3$, $\mathfrak{c}_2 = \mathfrak{c}$

$\mathfrak{a}' = \mathfrak{b}'''$, $\mathfrak{b}' = \mathfrak{b}$, $\mathfrak{c}' = \mathfrak{c}+\mathfrak{c}_3$; $\mathfrak{a}_1 = \mathfrak{c}_3$, $\mathfrak{b}_1 = \mathfrak{b}-\mathfrak{b}'''$, $\mathfrak{c}_1 = \mathfrak{c}$

$\mathfrak{a}_0 = \mathfrak{c}+\mathfrak{c}_3$, $\mathfrak{b}_0 = \mathfrak{b}_2-\mathfrak{b}'''$, $\mathfrak{c}_0 = \mathfrak{c}+\mathfrak{c}_3$
$= \mathfrak{b}-\mathfrak{b}'''$

$(\mathfrak{c}_3\mathfrak{b}) = 1 = h\mathfrak{c}\mathfrak{b}$; $h = 1$, $f = 1$, $\mathfrak{b}_1 = 1$

$\mathfrak{a}'' = \mathfrak{a}'$	$\mathfrak{a}_2 = \mathfrak{a}_1$
$\mathfrak{b}'' = \mathfrak{b}'$	$\mathfrak{b}_2 = \mathfrak{b}_1$
$\mathfrak{c}'' = \mathfrak{c}'$	$\mathfrak{c}_2 = \mathfrak{c}_1$

$\mathfrak{d}''' = \mathfrak{c}'''$, $\mathfrak{a}'' = \mathfrak{b}'''$, $\mathfrak{b}''' = \mathfrak{a}''$, $\mathfrak{c}'' = \mathfrak{d}'$, $\mathfrak{c}' = \mathfrak{c}_0$, $\mathfrak{c} = \mathfrak{c}_1$
~~[illegible]~~
$\mathfrak{b}_3 = \mathfrak{d}_4$; $\mathfrak{a}_2 = \mathfrak{c}_3$, $\mathfrak{c}_2 = \mathfrak{a}_3$, $\mathfrak{d}_1 = \mathfrak{b}_2$, $\mathfrak{b}_0 = \mathfrak{b}_1$, $\mathfrak{b}' = \mathfrak{b}$

$\mathfrak{d}' = \mathfrak{a}_2 = \mathfrak{b}_0 = \mathfrak{c}_0 = \mathfrak{d}_1$

Also

$\mathfrak{d}' = \mathfrak{a}_0 = \mathfrak{b}_0 = \mathfrak{c}_0 = \mathfrak{d}_1 = \mathfrak{c}'' = \mathfrak{c}' = \mathfrak{b}_2 = \mathfrak{b}_1$; $\mathfrak{d}''' = \mathfrak{c}'''$; $\mathfrak{d}_4 = \mathfrak{b}_3$

$\mathfrak{a}'' = \mathfrak{a}' = \mathfrak{b}'''$; $\mathfrak{a}_2 = \mathfrak{a}_1 = \mathfrak{c}_3$

$\mathfrak{b}'' = \mathfrak{b}' = \mathfrak{b}$; $\mathfrak{c}_2 = \mathfrak{c}_1 = \mathfrak{c} = \mathfrak{a}_3$
$= \mathfrak{a}'''$

Also bleiben 8 Moduln

$\mathfrak{c}'''$; $\mathfrak{b}, \mathfrak{a}'$; ~~[illegible]~~ $\mathfrak{a}_0$; $\mathfrak{a}, \mathfrak{c}$; $\mathfrak{a}_1$; $\mathfrak{c}_3$

	X' $\mathfrak{c}'''$	X $\mathfrak{b}$	Ψ' $\mathfrak{b}'$	H $\mathfrak{a}$	$X\Psi'$ $\mathfrak{a}_2$	Ψ $\mathfrak{c}$	HX $\mathfrak{a}_1$	$H\Psi$ $\mathfrak{b}_3$
X'		X	Ψ'	H	$X\Psi'$	Ψ	HX	$H\Psi$
X	X'		$X\Psi'$	HX	$X\Psi'$	Ψ	HX	$H\Psi$
Ψ'	X'	X'		H	$X\Psi'$	Ψ	HX	$H\Psi$
H	X'	X'	Ψ'		HX	$H\Psi$	HX	$H\Psi$
$X\Psi'$	X'	X	Ψ'	Ψ'		Ψ	HX	$H\Psi$
Ψ	X'	X	Ψ'	Ψ'	$X\Psi'$		$H\Psi$	$H\Psi$
HX	X'	X	Ψ'	H	$X\Psi'$	$X\Psi'$		$H\Psi$
$H\Psi$	X'	X	Ψ'	H	$X\Psi'$	Ψ	XH	

Ψ, Φ, Π

a. g. z. g.

$X' < X$; Ψ'
$X < X\Psi'$
$\Psi' < X\Psi'$, H
$X\Psi' < HX$, Ψ
$H < HX$
$HX < H\Psi$
$\Psi < H\Psi$

$\mathfrak{c}+(\mathfrak{a}-\mathfrak{b})$
$= (\mathfrak{c}+\mathfrak{a})-\mathfrak{b}$
$\mathfrak{c}''' < \mathfrak{a}', \mathfrak{b}$
$\mathfrak{a}' < \mathfrak{a}, \mathfrak{a}_2$
$\mathfrak{b} < \mathfrak{a}_2$
$\mathfrak{a} < \mathfrak{a}_1$
$\mathfrak{a}_2 < \mathfrak{a}_1, \mathfrak{c}$
$\mathfrak{a}_1 < \mathfrak{b}_3$
$\mathfrak{c} < \mathfrak{b}_3$

$1 < X < \Psi < \Phi$ und X Normaltheiler von Ψ

$1 < H < \Phi$ und H Normaltheiler von Φ.

$\Psi' = H - \Psi$; ~~[illegible]~~ Normaltheiler von Ψ

$X' = H + X = X + \Psi'$ Normaltheiler von Ψ

$H - \Psi = H\Psi$

$~~[illegible]~~$

$H - X = HX$

$X - \Psi' = X\Psi' = HX + \Psi$; in der That gilt hier das Modulgesetz $X - (H+\Psi') = (X-H)+\Psi$

$\mathfrak{N}'$: Theiler von XH und $X\Psi = \Psi$
$= X$ [Theiler von H und $X\Psi = \Psi$],
$= X\Psi'$

$\mathfrak{c}'' = (\Psi, H) = (\Psi', H) = (\Psi, H\Psi)$	$\mathfrak{c}' = (H\Psi, \Phi)$
$\mathfrak{f}' = (X', \Psi') = (X, \Psi') = (X, X\Psi')$	$\mathfrak{f}' = (X\Psi', \Psi) = (HX+\Psi, \Psi) = (HX, \Psi) = (HX, H\Psi)$
$\mathfrak{g}' = (1, X')$	$\mathfrak{g}' = (X', X) = (\Psi', X) = (\Psi', X\Psi') = (H, X) = (H, HX)$

$\boxed{\Phi = \mathfrak{M}\ ;\ X = \mathfrak{M}'\ ;\ \Psi = \mathfrak{P}}$

m natürliche Zahl

$\mathfrak{M}$ Gruppe der $\varphi(m)$ Classen (mod. m), welche relative Primzahlen zu m enthalten

$\mathfrak{H}$ eine in $\mathfrak{M}$ enthaltene Gruppe (Theiler von $\mathfrak{M}$)

$(\mathfrak{H}, \mathfrak{M}) = n$ Index von $\mathfrak{H}$ nach $\mathfrak{M}$; $\varphi(m) = nn'$; $n' = (1, \mathfrak{H})$

p eine natürliche Primzahl

p' die höchste in m aufgehende Potenz von p

$m = m'p'$; m' der grösste durch p nicht theilbare Divisor von m.

$\mathfrak{M}'$ ~~derjenige Theiler von M, welcher~~ Inbegriff derjenigen Classen (mod. m) in $\mathfrak{M}$, deren Zahlen $\equiv 1$ (mod. m')

$\mathfrak{P}$ Inbegriff derjenigen Classen (mod. m) in $\mathfrak{M}$, deren Zahlen $\equiv 1, p, p^2, p^3 \ldots$ (mod. m')

$(1, \mathfrak{M}') = \varphi(p')$; $(\mathfrak{M}', \mathfrak{M}) = \varphi(m')$ | Erst $\mathfrak{P}$, dann $\mathfrak{M}'$ definiren; $\mathfrak{M}'$ Theiler von $\mathfrak{P}$ |

$e_0 = (\mathfrak{P}, \mathfrak{M})$, $f_0 = (\mathfrak{M}', \mathfrak{P})$, $g_0 = (1, \mathfrak{M}') = \varphi(p')$; $e_0 f_0 = \varphi(m')$

$\left\{\begin{array}{l}\mathfrak{A} + \mathfrak{B} \text{ gr. gem. Theiler der Gruppen } \mathfrak{A}, \mathfrak{B} \\ \mathfrak{A} - \mathfrak{B} = \mathfrak{A}\mathfrak{B} \text{ kl. gem. Vielfache der Gruppen } \mathfrak{A}, \mathfrak{B}\end{array}\right\}$ $(\mathfrak{A}, \mathfrak{B}) = (\mathfrak{A}+\mathfrak{B}, \mathfrak{B}) = (\mathfrak{A}, \mathfrak{A}-\mathfrak{B})$

$e = (\mathfrak{H}\mathfrak{P}, \mathfrak{M})$, $f = (\mathfrak{H}\mathfrak{M}', \mathfrak{H}\mathfrak{P})$, $g = (\mathfrak{H}, \mathfrak{H}\mathfrak{M}')$; $efg = n$

$f = (\mathfrak{H}-\mathfrak{M}', \mathfrak{H}-\mathfrak{P}) = (\mathfrak{H}-\mathfrak{M}', \mathfrak{P}) = (\mathfrak{P}+(\mathfrak{H}-\mathfrak{M}'), \mathfrak{P}) = ((\mathfrak{P}+\mathfrak{H})-\mathfrak{M}', \mathfrak{P})$

$g = (\mathfrak{H}, \mathfrak{H}-\mathfrak{M}') = (\mathfrak{H}, \mathfrak{M}') = (\mathfrak{H}+\mathfrak{M}', \mathfrak{M}')$

$1 < \mathfrak{M}' < \mathfrak{P} < \mathfrak{M}$; $1 < \mathfrak{H} < \mathfrak{M}$

$1 < \mathfrak{H}+\mathfrak{M}' < \mathfrak{H}+\mathfrak{P} < \mathfrak{H} < \mathfrak{H}-\mathfrak{M}' < \mathfrak{H}-\mathfrak{P} < \mathfrak{M}$

$(\mathfrak{H}, \mathfrak{M}') = (\mathfrak{H}+\mathfrak{M}', \mathfrak{M}') = (\mathfrak{H}, \mathfrak{H}-\mathfrak{M}')$; $(\mathfrak{H}, \mathfrak{P}) = (\mathfrak{H}+\mathfrak{P}, \mathfrak{P}) = (\mathfrak{H}, \mathfrak{H}-\mathfrak{P})$

$(\mathfrak{M}', \mathfrak{H}) = (\mathfrak{H}+\mathfrak{M}', \mathfrak{H}) = (\mathfrak{M}', \mathfrak{H}-\mathfrak{M}')$; $(\mathfrak{P}, \mathfrak{H}) = (\mathfrak{H}+\mathfrak{P}, \mathfrak{H}) = (\mathfrak{P}, \mathfrak{H}-\mathfrak{P})$

$\mathfrak{H}+\mathfrak{M}' = \mathfrak{H}''$, $\mathfrak{H}-\mathfrak{M}' = \mathfrak{H}_1$ | $\mathfrak{H}_0 = \mathfrak{P}+(\mathfrak{H}-\mathfrak{M}') = (\mathfrak{H}+\mathfrak{P})-\mathfrak{M}'$

$\mathfrak{H}+\mathfrak{P} = \mathfrak{H}'$; $\mathfrak{H}-\mathfrak{P} = \mathfrak{H}_2$ | $\mathfrak{H}_0 = \mathfrak{P}+\mathfrak{H}_1 = \mathfrak{H}'-\mathfrak{M}'$

$1 < \mathfrak{H}''$; $\mathfrak{H}'' < \mathfrak{H}', \mathfrak{M}'$;

$\mathfrak{H}' < \mathfrak{H}, \mathfrak{H}_0$

$\mathfrak{M}' < \mathfrak{H}_0$

$\mathfrak{H} < \mathfrak{H}_1$

$\mathfrak{H}_0 < \mathfrak{H}_1, \mathfrak{P}$

$\mathfrak{H}_1 < \mathfrak{H}_2$

$\mathfrak{P} < \mathfrak{H}_2$

$\mathfrak{H}_2 < \mathfrak{M}$

$\left\{\begin{array}{l} f = (\mathfrak{H}_0, \mathfrak{P}) = (\mathfrak{H}_1, \mathfrak{H}_2) = (\mathfrak{H}, \mathfrak{P}) \\ g = (\mathfrak{H}, \mathfrak{H}_1) = (\mathfrak{H}'', \mathfrak{M}') = (\mathfrak{H}, \mathfrak{M}') \\ e = (\mathfrak{H}_2, \mathfrak{M}) \end{array}\right\}$

$\left\{\begin{array}{l} e_0 = (\mathfrak{P}, \mathfrak{M}) = (\mathfrak{P}, \mathfrak{H}_2)(\mathfrak{H}_2, \mathfrak{M}) = e\,(\mathfrak{P}, \mathfrak{H}_2) \\ f_0 = (\mathfrak{M}', \mathfrak{P}) = (\mathfrak{M}', \mathfrak{H}_0)(\mathfrak{H}_0, \mathfrak{P}) = f\,(\mathfrak{M}', \mathfrak{H}_0) \\ g_0 = (1, \mathfrak{M}') = (1, \mathfrak{H}'')(\mathfrak{H}'', \mathfrak{M}') = g\,(1, \mathfrak{H}'') \end{array}\right\}$

$\left\{\begin{array}{l} e' = (\mathfrak{P}, \mathfrak{H}_2) = (\mathfrak{P}, \mathfrak{H}-\mathfrak{P}) = (\mathfrak{P}, \mathfrak{H}) = (\mathfrak{H}', \mathfrak{H}) \\ f' = (\mathfrak{M}', \mathfrak{H}_0) = (\mathfrak{M}', \mathfrak{H}'-\mathfrak{M}') = (\mathfrak{M}', \mathfrak{H}') = (\mathfrak{H}'', \mathfrak{H}') \\ g' = (1, \mathfrak{H}'') \end{array}\right\}$ $\begin{array}{l} e_0 = ee' \\ f_0 = ff' \\ g_0 = gg' \end{array}$ | $\begin{array}{l} e_0 f_0 g_0 = \varphi(m) \\ efg = n \\ e'f'g' = n' \end{array}$

$g_0 = \varphi(p')$; $e_0 f_0 = \varphi(m')$

$\mathfrak{o}p = (\mathfrak{y}_1 \mathfrak{y}_2 \cdots \mathfrak{y}_e)^g$; $N(\mathfrak{y}_r) = p^f$

$\mathfrak{z}\mathfrak{y}_r = (\mathfrak{y}_{r,1}\, \mathfrak{y}_{r,2} \cdots \mathfrak{y}_{r,e'})^{g'}$; $\pi(\mathfrak{y}_{r,s}) = \mathfrak{y}_r^{f'}$; $\pi(\mathfrak{y}_r) = \mathfrak{y}_r^{e'f'g'} = \mathfrak{y}_r^{n'}$

$\mathfrak{z}p = \prod^{e}_{r} \mathfrak{y}_{r,s}^{gg'} (\prod^{e} \mathfrak{y}_{r,s})^{g}$ $\quad N_0(\mathfrak{y}_{r,s}) = N\pi(\mathfrak{y}_{r,s}) = N(\mathfrak{y}_r^{f'}) = p^{ff'} = p^{f_0}$