# Théorie des 3 modules, grand tableau et nombres de classes

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En passant la souris sur une vignette, le titre de l'image apparaît.

2 Fichier(s)

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## Présentation

TitreThéorie des 3 modules, grand tableau et nombres de classes
Date 189x
Sujet

- dualisme
- dualité
- nombres de classes
- notation3
- trois modules

CoteCod. Ms. Dedekind X 11-2, p. 15
Format1 p. ; 2 f.
LangueAllemand

## Description & Analyse

DescriptionGrand tableau PGCD/PPCM avec notation3 et détail des définitions. Etude de la dualité dans les nombres de classes.
Mode(s) d'écriture

- Document de travail
- Tableau

## Relations

**Collection Cod. Ms. Dedekind X 11-1**

*Ce document utilise la même notation que :*

 [Théorie des trois modules, divisibilité.]

Afficher la visualisation des relations de la notice.

## Mots-clefs

dualisme, dualite, nombres de classes, notation3, trois modules
Notice créée par Emmylou Haffner Notice créée le 19/07/2021 Dernière modification le 21/07/2021

# Grösste gemeinschaftliche Theiler.

Theorie der [illegible] Moduln

| | $d'''$ | $a''$ $b''$ $c''$ | $a'$ $b'$ $c'$ | $d'$ $a'$ $b'$ $c'$ | $a$ $b$ $c$ $a_0$ $b_0$ $c_0$ | $d_1$ $a_1$ $b_1$ $c_1$ | $a_2$ $b_2$ $c_2$ | $a_3$ $b_3$ $c_3$ | $d_4$ | + |
|---|---|---|---|---|---|---|---|---|---|---|
| $d'''$ | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | $d'''$ |
| $a'''$ | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | $a'''$ |
| $b'''$ | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | $b'''$ |
| $c'''$ | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | $c'''$ |
| $a''$ | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | $a''$ |
| $b''$ | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | $b''$ |
| $c''$ | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | $c''$ |
| $d'$ | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | $d'$ |
| $a'$ | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | $a'$ |
| $b'$ | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | $b'$ |
| $c'$ | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | $c'$ |
| $a$ | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | $a$ |
| $b$ | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | $b$ |
| $c$ | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | $c$ |
| $a_0$ | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | $a_0$ |
| $b_0$ | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | $b_0$ |
| $c_0$ | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | $c_0$ |
| $d_1$ | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | $d_1$ |
| $a_1$ | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | $a_1$ |
| $b_1$ | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | $b_1$ |
| $c_1$ | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | $c_1$ |
| $a_2$ | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | $a_2$ |
| $b_2$ | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |
| $c_2$ | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |
| $a_3$ | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |
| $b_3$ | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |
| $c_3$ | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |
| $d_4$ | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |
| — | $d'''$ | $a''$ $b''$ $c''$ | $a'$ $b'$ $c'$ | $d'$ $a'$ $b'$ $c'$ | $a$ $b$ $c$ $a_0$ $b_0$ $c_0$ | $d_1$ $a_1$ $b_1$ $c_1$ | $a_2$ $b_2$ $c_2$ | $a_3$ $b_3$ $c_3$ | $d_4$ | |

# Kleinste gemeinschaftliche Vielfache.

## Definitionen:

$d''' = a+b+c$ ; $d_4 = a-b-c$

$a''' = b+c$, $b''' = c+a$, $c''' = a+b$ ; $a_3 = b-c$, $b_3 = c-a$, $c_3 = a-b$

$d' = a'''-b'''-c'''$ ; $d_1 = a_3+b_3+c_3$

$a'' = a+d'$, $b'' = b+d'$, $c'' = c+d'$ ; $a_2 = a-d_1$, $b_2 = b-d_1$, $c_2 = c-d_1$

$a' = a+d_1$, $b' = b+d_1$, $c' = c+d_1$ ; $a_1 = a-d'$, $b_1 = b-d'$, $c_1 = c-d'$

$a_0 = a'-d' = a_1+d_1$, $b_0 = b'-d' = b_1+d_1$, $c_0 = c'-d' = c_1+d_1$

Oder auch:

$a'' = b'''-c'''$, $b'' = c'''-a'''$, $c'' = a'''-b'''$ ; $a_2 = b_3+c_3$, $b_2 = c_3+a_3$, $c_2 = a_3+b_3$

$a' = a+a_3$, $b' = b+b_3$, $c' = c+c_3$ ; $a_1 = a-a'''$, $b_1 = b-b'''$, $c_1 = c-c'''$

$a_0 = a_1+a_3 = a'-a'''$, $b_0 = b_1+b_3 = b'-b'''$, $c_0 = c_1+c_3 = c'-c'''$

Roth unterstrichen sind die 2. 117 Resultanten, welche von jeder [illegible] [illegible] die übrigen 2. 261 Stellen geben Theilbarkeiten. 261 + 117 = 378 = [illegible] [illegible]

## Einfache Schritte und Klassen-Anzahlen:

$$(\mathfrak{d}''', \mathfrak{a}''') = (\mathfrak{b}'', \mathfrak{c}'') = (\mathfrak{c}'', \mathfrak{b}'') = (\mathfrak{a}'', \mathfrak{d}') = (\mathfrak{a}', \mathfrak{a}_0) = (\mathfrak{a}, \mathfrak{a}_1) = a$$
$$(\mathfrak{d}''', \mathfrak{b}''') = (\mathfrak{c}'', \mathfrak{a}'') = (\mathfrak{a}''', \mathfrak{c}'') = (\mathfrak{b}'', \mathfrak{d}') = (\mathfrak{b}', \mathfrak{b}_0) = (\mathfrak{b}, \mathfrak{b}_1) = b$$
$$(\mathfrak{d}''', \mathfrak{c}''') = (\mathfrak{a}'', \mathfrak{b}'') = (\mathfrak{b}'', \mathfrak{a}'') = (\mathfrak{c}'', \mathfrak{d}') = (\mathfrak{c}', \mathfrak{c}_0) = (\mathfrak{c}, \mathfrak{c}_1) = c$$
$$(\mathfrak{a}_3, \mathfrak{d}_4) = (\mathfrak{c}_2, \mathfrak{b}_3) = (\mathfrak{b}_2, \mathfrak{c}_3) = (\mathfrak{d}_1, \mathfrak{a}_2) = (\mathfrak{a}_0, \mathfrak{a}_1) = (\mathfrak{a}', \mathfrak{a}) = a_1$$
$$(\mathfrak{b}_3, \mathfrak{d}_4) = (\mathfrak{a}_2, \mathfrak{c}_3) = (\mathfrak{c}_2, \mathfrak{a}_3) = (\mathfrak{d}_1, \mathfrak{b}_2) = (\mathfrak{b}_0, \mathfrak{b}_1) = (\mathfrak{b}', \mathfrak{b}) = b_1$$
$$(\mathfrak{c}_3, \mathfrak{d}_4) = (\mathfrak{b}_2, \mathfrak{a}_3) = (\mathfrak{a}_2, \mathfrak{b}_3) = (\mathfrak{d}_1, \mathfrak{c}_2) = (\mathfrak{c}_0, \mathfrak{c}_1) = (\mathfrak{c}', \mathfrak{c}) = c_1$$

$$\left.\begin{array}{l}(\mathfrak{a}'', \mathfrak{a}') = (\mathfrak{b}'', \mathfrak{b}') = (\mathfrak{c}'', \mathfrak{c}') = (\mathfrak{d}', \mathfrak{a}_0) = (\mathfrak{d}', \mathfrak{b}_0) = (\mathfrak{d}', \mathfrak{c}_0) = \\ (\mathfrak{a}_1, \mathfrak{a}_2) = (\mathfrak{b}_1, \mathfrak{b}_2) = (\mathfrak{c}_1, \mathfrak{c}_2) = (\mathfrak{a}_0, \mathfrak{d}_1) = (\mathfrak{b}_0, \mathfrak{d}_1) = (\mathfrak{c}_0, \mathfrak{d}_1) = \end{array}\right\} = h \quad \underline{\text{[illegible]}}$$

$$\begin{array}{l|l|l} (\mathfrak{b}, \mathfrak{c}) = h b c_1 & (\mathfrak{c}, \mathfrak{b}) = h c b_1 & (\mathfrak{d}''', \mathfrak{d}') = abc \\ (\mathfrak{c}, \mathfrak{a}) = h c a_1 & (\mathfrak{a}, \mathfrak{c}) = h a c_1 & (\mathfrak{d}', \mathfrak{d}_1) = h^2 \\ (\mathfrak{a}, \mathfrak{b}) = h a b_1 & (\mathfrak{b}, \mathfrak{a}) = h b a_1 & (\mathfrak{d}_1, \mathfrak{d}_4) = a_1 b_1 c_1 \end{array}$$

$$(\mathfrak{b}, \mathfrak{c})(\mathfrak{c}, \mathfrak{a})(\mathfrak{a}, \mathfrak{b}) = (\mathfrak{c}, \mathfrak{b})(\mathfrak{a}, \mathfrak{c})(\mathfrak{b}, \mathfrak{a}) = h^3 abc\, a_1 b_1 c_1 = h(\mathfrak{d}''', \mathfrak{d}_4)$$

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Den letzten Satz kann man auch ohne die vollständige Theorie der drei Moduln (Dualismus), [illegible] durch die drei Elementar-Sätze

I. $(\mathfrak{a}, \mathfrak{b}) = (\mathfrak{a}+\mathfrak{b}, \mathfrak{b})$.

II. $(\mathfrak{a}, \mathfrak{b}) = (\mathfrak{a}, \mathfrak{a}-\mathfrak{b})$.

III. Wenn $\mathfrak{y}$ Divisor von $\mathfrak{m}$, und $\mathfrak{m}$ Divisor von $\mathfrak{w}$, so ist $(\mathfrak{y}, \mathfrak{w}) = (\mathfrak{y}, \mathfrak{m})(\mathfrak{m}, \mathfrak{w})$

beweisen. In der That, mit Anwendung der obigen Bezeichnungen (nicht Sätze) ist

$(\mathfrak{b}, \mathfrak{c}) = (\mathfrak{b}+\mathfrak{c}, \mathfrak{c}) = (\mathfrak{a}'', \mathfrak{c})$ zufolge I,

da ferner $\mathfrak{a}'' - \mathfrak{b}'''$ Multiplum von $\mathfrak{a}''$ und Divisor von $\mathfrak{c}$ ist, so ist

$(\mathfrak{a}'', \mathfrak{c}) = (\mathfrak{a}''', \mathfrak{c}'')(\mathfrak{c}'', \mathfrak{c})$ zufolge III;

ferner ist

$(\mathfrak{a}''', \mathfrak{c}'') = (\mathfrak{a}''', \mathfrak{a}''' - \mathfrak{b}''') = (\mathfrak{a}''', \mathfrak{b}''')$ zufolge II,

es ist aber ~~[illegible]~~ $\mathfrak{a}''' + \mathfrak{b}''' = (\mathfrak{b}+\mathfrak{c}) + (\mathfrak{c}+\mathfrak{a}) = \mathfrak{b}+\mathfrak{c}+\mathfrak{a} = \mathfrak{b}+\mathfrak{b}'''$, mithin

$(\mathfrak{a}''', \mathfrak{b}''') = (\mathfrak{a}''' + \mathfrak{b}''', \mathfrak{b}''') = (\mathfrak{b}+\mathfrak{b}''', \mathfrak{b}''') = (\mathfrak{b}, \mathfrak{b}''')$ zufolge I,

und folglich ergiebt sich die erste der sechs Gleichungen

$$\text{IV.} \begin{cases} (\mathfrak{b}, \mathfrak{c}) = (\mathfrak{b}, \mathfrak{b}''')(\mathfrak{c}'', \mathfrak{c});\ (\mathfrak{c}, \mathfrak{a}) = (\mathfrak{c}, \mathfrak{c}''')(\mathfrak{a}'', \mathfrak{a});\ (\mathfrak{a}, \mathfrak{b}) = (\mathfrak{a}, \mathfrak{a}''')(\mathfrak{b}'', \mathfrak{b}) \\ (\mathfrak{c}, \mathfrak{b}) = (\mathfrak{c}, \mathfrak{c}''')(\mathfrak{b}'', \mathfrak{b});\ (\mathfrak{a}, \mathfrak{c}) = (\mathfrak{a}, \mathfrak{a}''')(\mathfrak{c}'', \mathfrak{c});\ (\mathfrak{b}, \mathfrak{a}) = (\mathfrak{b}, \mathfrak{b}''')(\mathfrak{a}'', \mathfrak{a}) \end{cases}$$

woraus die übrigen durch Permutation folgen; und hieraus ergiebt sich der obige Satz

V. $(\mathfrak{b}, \mathfrak{c})(\mathfrak{c}, \mathfrak{a})(\mathfrak{a}, \mathfrak{b}) = (\mathfrak{c}, \mathfrak{b})(\mathfrak{a}, \mathfrak{c})(\mathfrak{b}, \mathfrak{a}) = (\mathfrak{a}, \mathfrak{a}''')(\mathfrak{b}, \mathfrak{b}''')(\mathfrak{c}, \mathfrak{c}''')(\mathfrak{a}'', \mathfrak{a})(\mathfrak{b}'', \mathfrak{b})(\mathfrak{c}'', \mathfrak{c})$

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Ähnlicherweise ergiebt sich ebenso

$(\mathfrak{b}, \mathfrak{c}) = (\mathfrak{b}, \mathfrak{b}+\mathfrak{c}) = (\mathfrak{b}, \mathfrak{a}_3)$ zufolge I,

da ferner $\mathfrak{b}_2 = \mathfrak{c}_2 + \mathfrak{a}_3$ Divisor von $\mathfrak{a}_3$ und Multiplum von $\mathfrak{b}$ ist, so ist

$(\mathfrak{b}, \mathfrak{a}_3) = (\mathfrak{b}, \mathfrak{b}_2)(\mathfrak{b}_2, \mathfrak{a}_3)$ zufolge III,

ferner ist

$(\mathfrak{b}_2, \mathfrak{a}_3) = (\mathfrak{c}_3 + \mathfrak{a}_3, \mathfrak{a}_3) = (\mathfrak{c}_3, \mathfrak{a}_3)$ zufolge I,

es ist aber $\mathfrak{c}_3 - \mathfrak{a}_3 = (\mathfrak{a}-\mathfrak{b}) - (\mathfrak{b}-\mathfrak{c}) = \mathfrak{a}-\mathfrak{b}-\mathfrak{c} = \mathfrak{c}-\mathfrak{c}_3$, mithin

$(\mathfrak{c}_3, \mathfrak{a}_3) = (\mathfrak{c}_3, \mathfrak{c}_3 - \mathfrak{a}_3) = (\mathfrak{c}_3, \mathfrak{c} - \mathfrak{c}_3) = (\mathfrak{c}_3, \mathfrak{c})$ zufolge II,

woraus

$$\text{VI.} \begin{cases} (\mathfrak{b}, \mathfrak{c}) = (\mathfrak{b}, \mathfrak{b}_2)(\mathfrak{c}_3, \mathfrak{c});\ (\mathfrak{c}, \mathfrak{a}) = (\mathfrak{c}, \mathfrak{c}_2)(\mathfrak{a}_3, \mathfrak{a});\ (\mathfrak{a}, \mathfrak{b}) = (\mathfrak{a}, \mathfrak{a}_2)(\mathfrak{b}_3, \mathfrak{b}) \\ (\mathfrak{c}, \mathfrak{b}) = (\mathfrak{c}, \mathfrak{c}_2)(\mathfrak{b}_3, \mathfrak{b});\ (\mathfrak{a}, \mathfrak{c}) = (\mathfrak{a}, \mathfrak{a}_2)(\mathfrak{c}_3, \mathfrak{c});\ (\mathfrak{b}, \mathfrak{a}) = (\mathfrak{b}, \mathfrak{b}_2)(\mathfrak{a}_3, \mathfrak{a}) \end{cases}$$

und

VII. $(\mathfrak{b}, \mathfrak{c})(\mathfrak{c}, \mathfrak{a})(\mathfrak{a}, \mathfrak{b}) = (\mathfrak{c}, \mathfrak{b})(\mathfrak{a}, \mathfrak{c})(\mathfrak{b}, \mathfrak{a}) = (\mathfrak{a}, \mathfrak{a}_2)(\mathfrak{b}, \mathfrak{b}_2)(\mathfrak{c}, \mathfrak{c}_2)(\mathfrak{a}_3, \mathfrak{a})(\mathfrak{b}_3, \mathfrak{b})(\mathfrak{c}_3, \mathfrak{c})$