Trois modules (ou groupes abéliens)

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En passant la souris sur une vignette, le titre de l'image apparaît.

2 Fichier(s)

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Présentation

TitreTrois modules (ou groupes abéliens)
Date 189x
Sujet

- groupes
- Modulgesetz
- trois modules

CoteCod. Ms. Dedekind X 11-2, p. 14
Format1 p., 2 f.
LangueAllemand

Description & Analyse

DescriptionCommence par une étude du "groupe" généré par 3 modules ou trois groupes abéliens. Reformulation dans la notation utilisée pour la théorie des groupes (eg Modulgesetz). Étude du treillis formé par les sous-groupes normaux.
NotesRédigé par dessus une lettre.
Mode(s) d'écriture

- Calculs
- Tableau

Auteur·es de la descriptionHaffner, Emmylou

Relations

Ce document n'a pas de relation indiquée avec un autre document du projet.

Mots-clefs

Groupes, Modulgesetz, trois modules
Notice créée par Emmylou Haffner Notice créée le 19/07/2021 Dernière modification le 21/07/2021

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Drei Moduln (oder Modulsche Systeme) a, b, c mit der Summe $d < c$, a, b

$a''' = b + c = b$; $a_3 = b - c = c$

$a''' = b$; $b''' = c + a$; $c''' = a + b$; $a_2 = c$; $b_2 = c - a$; $c_2 = a - b$

$d''' = c'''$, $d_4 = b_2$, $d' = b - b'''$; $d_1 = c + c_2$

$a'' = b'''$, $b'' = b$, $c'' = b - b'''$; $a_2 = c_2$, $b_2 = c + c_2$; $c_2 = c$

$a' = b'''$, $b' = b$, $c' = c + c_2$; $a_1 = c_2$, $b_1 = b - b'''$, $c_1 = c$

$a_0 = c + c_2$, $b_0 = b_2 - b'''$, $c_0 = c + c_2$

$= b - b'''$

$(c, b) = 1 = h c b_2$; $h = 1$; $f_1 = 1$; $b_1 = 1$

$a'' = a'$ | $a_2 = a_1$; $b'' = b'$ | $b_2 = b_1$; $c'' = c'$ | $c_2 = c_1$; $d''' = c'''$, $a''' = b''$, $b''' = a''$, $c'' = d'$, $c' = c_0$, $c = c_1$

$b_3 = d_4$, $a_2 = c_2$, $c_2 = a_3$, $d_1 = b_2$, $b_0 = b_1$, $b' = b$

$d' = a_2 = b_0 = c_3 = d_1$

Also

$d' = a_2 = b_0 = c_3 = d_1 = c'' = c' = b_2 = b_1$; $d''' = c'''$; $d_4 = b_3$

$a'' = a' = b'''$; $a_2 = a_1 = c_3$

$b'' = b' = b$; $c_2 = c_1 = c = a_3$

$= a'''$

Also bleiben 8 Moduln

c''' ; b , a' ; a_0 ; a_2 ; a_1 ; c_3

	X' c'''	X $\mathfrak{b}$	Ψ' $\mathfrak{a}'$	H $\mathfrak{a}$	$X\Psi'$ $\mathfrak{a}_0$	Ψ $\mathfrak{c}$	HX $\mathfrak{a}_1$	$H\Psi$ $\mathfrak{b}_3$
X'		X	Ψ'	H	$X\Psi'$	Ψ	HX	$H\Psi$
X	X'		$X\Psi'$	HX	$X\Psi'$	Ψ	HX	$H\Psi$
Ψ'	X'	X'		H	$X\Psi'$	Ψ'	HX	$H\Psi$
H	X'	X'	Ψ'		HX	$H\Psi$	HX	$H\Psi$
$X\Psi'$	X'	X'	Ψ'	Ψ'		Ψ	HX	$H\Psi$
Ψ	X'	X'	Ψ'	Ψ'	$X\Psi'$		$H\Psi$	$H\Psi$
HX	X'	X'	Ψ'	H	$X\Psi'$	$X\Psi'$		$H\Psi$
$H\Psi$	X'	X'	Ψ'	H	$X\Psi'$	Ψ	XH	

$X' < X$; Ψ' ...

$X < X\Psi'$

$\Psi' < X\Psi'$, H

$X\Psi' < HX$, Ψ

$H < HX$ ...

$HX < H\Psi$

$\Psi < H\Psi$

$1 < X < \Psi < \bar{\Psi}$ und X Normalteiler von Ψ.

$1 < H < \Psi$ und H Normalteiler von $\bar{\Psi}$.

$\Psi' = H + \Psi$; Normalteiler von Ψ

$X' + H + X = X + \Psi'$ Normalteiler von Ψ

$H - \Psi = H\Psi$

$H - X = HX$

$X - \Psi' = X\Psi' = HX + \Psi$; in der That gilt hier das Modulgesetz $X - (H + \Psi') = (X - H) + \Psi$

$c'' = (X, H) = (\Psi', H) = (\Psi, H\Psi)$ | $c' = (H\Psi, \bar{\Psi})$

$f'' = (X', \Psi') = (X, \Psi') = (X, X\Psi')$ | $f' = (X\Psi', \Psi) = (HX + \Psi, \Psi) = (HX, \Psi) = (HX, H\Psi)$

$g'' = (\Psi, X')$ | $g' = (X', X) = (\Psi', X) = (\Psi', X\Psi') = (H, X) = (H, HX)$

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$\mathfrak{H} = \mathfrak{M}$, $\mathfrak{X} = \mathfrak{M}'$, $\mathfrak{Y} = \mathfrak{P}$

m natürliche Zahl

$\mathfrak{M}$ Gruppe der $\varphi(m)$ Klassen (mod. m) , welche relativ Primzahlen zu m enthalten

$\mathfrak{H}$ eine in $\mathfrak{M}$ enthaltene Gruppe (Theiler von $\mathfrak{M}$)

$(\mathfrak{H}, \mathfrak{M}) = n$ Index von $\mathfrak{H}$ in $\mathfrak{M}$; $\varphi(m) = nn'$, $\mathfrak{n}' = (\mathfrak{e}, \mathfrak{H})$

p eine natürliche Primzahl

p^λ die höchste in m aufgehende Potenz von p

$m = m' p^\lambda$, m' der größte durch p nicht theilbare Theiler von m ,

$\mathfrak{M}'$ diejenige Theilergruppe Inbegriff derjenigen Klassen (mod. m) in $\mathfrak{M}$, deren Zahlen $\equiv 1$ (mod. m')

$\mathfrak{P}$ Inbegriff derjenigen Klassen (mod. m) in $\mathfrak{M}$, deren Zahlen $\equiv 1, p, p^2, p^3 \cdots$ (mod. m')

$(\mathfrak{e}, \mathfrak{M}') = \varphi(p^\lambda)$, $(\mathfrak{M}', \mathfrak{M}) = \varphi(m')$ | Ist $\mathfrak{P}$, dann $\mathfrak{M}'$ desgleichen, $\mathfrak{M}'$ Theiler von $\mathfrak{P}$ |

$\mathfrak{e}_0 = (\mathfrak{P}, \mathfrak{M})$, $\mathfrak{f}_0 = (\mathfrak{M}', \mathfrak{P})$, $\mathfrak{g}_0 = (\mathfrak{e}, \mathfrak{M}') = \varphi(p^\lambda)$; $\mathfrak{e}_0 \mathfrak{f}_0 = \varphi(m')$

$\alpha + \beta$ gem. Theiler der Gruppen $\mathfrak{A}$, $\mathfrak{B}$
$\mathfrak{A} - \mathfrak{B} = \mathfrak{A} \mathfrak{B}$ kgv. Vielfache der Gruppen $\mathfrak{A}$, $\mathfrak{B}$
$(\mathfrak{A}, \mathfrak{B}) = (\mathfrak{A}+\mathfrak{B}, \mathfrak{B}) = (\mathfrak{A}, \mathfrak{A}-\mathfrak{B})$

$\mathfrak{e} = (\mathfrak{H}\mathfrak{P}, \mathfrak{M})$, $\mathfrak{f} = (\mathfrak{H}\mathfrak{M}', \mathfrak{H}\mathfrak{P})$, $\mathfrak{g} = (\mathfrak{H}, \mathfrak{H}\mathfrak{M}')$; $\mathfrak{e}\mathfrak{f}\mathfrak{g} = \mathfrak{n}$

$\mathfrak{f} = (\mathfrak{H}-\mathfrak{M}', \mathfrak{H}-\mathfrak{P}) = (\mathfrak{H}-\mathfrak{M}', \mathfrak{P}) = (\mathfrak{P}+(\mathfrak{H}-\mathfrak{M}'), \mathfrak{P}) = ((\mathfrak{P}+\mathfrak{H})-\mathfrak{M}', \mathfrak{P})$

$\mathfrak{g} = (\mathfrak{H}, \mathfrak{H}-\mathfrak{M}') = (\mathfrak{H}, \mathfrak{M}') = (\mathfrak{H}+\mathfrak{M}', \mathfrak{M}')$

$1 < \mathfrak{M}' < \mathfrak{P} < \mathfrak{M}$; $1 < \mathfrak{H} < \mathfrak{M}$

$1 < \mathfrak{H}+\mathfrak{M}' < \mathfrak{H}+\mathfrak{P} < \mathfrak{H} < \mathfrak{H}-\mathfrak{M}' < \mathfrak{H}-\mathfrak{P} < \mathfrak{M}$

$(\mathfrak{H}, \mathfrak{M}') = (\mathfrak{H}+\mathfrak{M}', \mathfrak{M}') = (\mathfrak{H}, \mathfrak{H}-\mathfrak{M}')$; $(\mathfrak{H}, \mathfrak{P}) = (\mathfrak{H}-\mathfrak{P}, \mathfrak{P}) = (\mathfrak{H}, \mathfrak{H}-\mathfrak{P})$

$(\mathfrak{M}', \mathfrak{H}) = (\mathfrak{H}+\mathfrak{M}', \mathfrak{H}) = (\mathfrak{M}', \mathfrak{H}-\mathfrak{M}')$; $(\mathfrak{P}, \mathfrak{H}) = (\mathfrak{H}+\mathfrak{P}, \mathfrak{H}) = (\mathfrak{P}, \mathfrak{H}-\mathfrak{P})$

$\mathfrak{H}+\mathfrak{M}' = \mathfrak{H}''$, $\mathfrak{H}-\mathfrak{M}' = \mathfrak{H}_1$
$\mathfrak{H}+\mathfrak{P} = \mathfrak{H}'$, $\mathfrak{H}-\mathfrak{P} = \mathfrak{H}_2$

$\mathfrak{K} = \mathfrak{P}+(\mathfrak{H}-\mathfrak{M}') = (\mathfrak{H}+\mathfrak{P})-\mathfrak{M}'$
$\mathfrak{K}_2 = \mathfrak{P}+\mathfrak{H}_1 = \mathfrak{H}'-\mathfrak{M}'$

$1 < \mathfrak{H}''$; $\mathfrak{H}'' < \mathfrak{H}', \mathfrak{M}'$;
$\mathfrak{H}' < \mathfrak{H}, \mathfrak{H}_2$
$\mathfrak{M}' < \mathfrak{H}_2$
$\mathfrak{H} < \mathfrak{H}_1$
$\mathfrak{H}_2 < \mathfrak{H}_1, \mathfrak{P}$
$\mathfrak{H}_1 < \mathfrak{K}_2$
$\mathfrak{P} < \mathfrak{H}_2$
$\mathfrak{H}_2 < \mathfrak{M}$

$\left\{ \begin{array}{l} \mathfrak{f} = (\mathfrak{H}_2, \mathfrak{P}) = (\mathfrak{H}_1, \mathfrak{H}_2) \\ \mathfrak{g} = (\mathfrak{H}, \mathfrak{H}_1) = (\mathfrak{H}'', \mathfrak{M}') \\ \mathfrak{e} = (\mathfrak{H}_2, \mathfrak{M}) \end{array} \right\} \left\{ \begin{array}{l} = (\mathfrak{H}_1, \mathfrak{P}) \\ = (\mathfrak{H}, \mathfrak{M}') \end{array} \right\}$

$\left\{ \begin{array}{l} \mathfrak{e}_0 = (\mathfrak{P}, \mathfrak{M}) = (\mathfrak{P}, \mathfrak{H}_2)(\mathfrak{H}_2, \mathfrak{M}) = \mathfrak{e}(\mathfrak{P}, \mathfrak{H}_2) \\ \mathfrak{f}_0 = (\mathfrak{M}', \mathfrak{P}) = (\mathfrak{M}', \mathfrak{H}_2)(\mathfrak{H}_2, \mathfrak{P}) = \mathfrak{f}(\mathfrak{M}', \mathfrak{H}_2) \\ \mathfrak{g}_0 = (1, \mathfrak{M}') = (1, \mathfrak{H}'')(\mathfrak{H}'', \mathfrak{M}') = \mathfrak{g}(1, \mathfrak{H}'') \end{array} \right\}$

$\left\{ \begin{array}{l} \mathfrak{e}' = (\mathfrak{P}, \mathfrak{H}_2) = (\mathfrak{P}, \mathfrak{H}-\mathfrak{P}) = (\mathfrak{P}, \mathfrak{H}) = (\mathfrak{H}', \mathfrak{H}) \\ \mathfrak{f}' = (\mathfrak{M}', \mathfrak{H}_2) = (\mathfrak{M}', \mathfrak{H}'-\mathfrak{M}') = (\mathfrak{M}', \mathfrak{H}') = (\mathfrak{H}'', \mathfrak{H}') \\ \mathfrak{g}' = (1, \mathfrak{H}'') \end{array} \right\}$

$\mathfrak{e}_0 = \mathfrak{e}\mathfrak{e}'$ | $\mathfrak{e}_0 \mathfrak{f}_0 \mathfrak{g}_0 = \varphi(m)$
$\mathfrak{f}_0 = \mathfrak{f}\mathfrak{f}'$ | $\mathfrak{e}\mathfrak{f}\mathfrak{g} = \mathfrak{n}$
$\mathfrak{g}_0 = \mathfrak{g}\mathfrak{g}'$ | $\mathfrak{e}'\mathfrak{f}'\mathfrak{g}' = \mathfrak{n}'$

$\mathfrak{g}_0 = \varphi(p^\lambda)$; $\mathfrak{e}_0 \mathfrak{f}_0 = \varphi(m')$

$\mathfrak{e}p = (g_1 g_2 \cdots g_e)^{\mathfrak{g}}$; $N(g_\varepsilon) = p^{\mathfrak{f}}$

$\mathfrak{g}_{\varepsilon+1} = (g_{\varepsilon+1,1}\, g_{\varepsilon+1,2} \cdots g_{\varepsilon+1,e'})^{\mathfrak{g}'}$; $\pi(g_{\varepsilon+1,\varepsilon}) = g_\varepsilon^{\mathfrak{f}'}$; $\pi(\mathfrak{g}_{\varepsilon+1}) = g_\varepsilon^{e'\mathfrak{f}'\mathfrak{g}'} = g_\varepsilon^{n'}$

$\mathfrak{e}_0 p = \overset{e_0}{\Pi} g_{\varepsilon,\varepsilon}^{\mathfrak{g}\mathfrak{g}'} \overset{e_0}{\Pi} g_{\varepsilon,1}^{\mathfrak{g}}$ $\qquad N_0(g_{\varepsilon,1}) = N\pi(g_{\varepsilon+1,\varepsilon}) = N(g_\varepsilon^{\mathfrak{f}'}) = p^{\mathfrak{f}\mathfrak{f}'} = p^{\mathfrak{f}_0}$

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