Théorie des 3 modules, grand tableau et nombres de classes

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En passant la souris sur une vignette, le titre de l'image apparaît.

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Présentation

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- dualité
- nombres de classes
- notation3
- trois modules

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Description & Analyse

DescriptionGrand tableau PGCD/PPCM avec notation3 et détail des définitions. Etude de la dualité dans les nombres de classes.
Mode(s) d'écriture

- Document de travail
- Tableau

Relations

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Ce document *utilise la même notation que :*

 Théorie des trois modules, divisibilité.

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Afficher la visualisation des relations de la notice.

Mots-clefs

dualisme, dualité, nombres de classes, notation3, trois modules
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Theorie der drei Moduln

Grösste gemeinschaftliche Theiler.

	$\mathfrak{d}'''$	a'''	b'''	c'''	a''	b''	c''	$\mathfrak{d}'$	a'	b'	c'	a	b	c	a_0	b_0	c_0	$\mathfrak{d}_1$	a_1	b_1	c_1	a_2	b_2	c_2	a_3	b_3	c_3	$\mathfrak{d}_4$	+
$\mathfrak{d}'''$	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
a'''	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
b'''	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
c'''	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
a''	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
b''	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
c''	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
$\mathfrak{d}'$	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
a'	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
b'	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
c'	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
a	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
b	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
c	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
a_0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
b_0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
c_0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
$\mathfrak{d}_1$	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
a_1	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
b_1	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
c_1	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
a_2	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
b_2	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
c_2	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
a_3	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
b_3	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
c_3	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
$\mathfrak{d}_4$	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
—	$\mathfrak{d}$	a''	b''	c''	a'	b'	c'	$\mathfrak{d}'$	a'	b'	c'	a	b	c	a_0	b_0	c_0	$\mathfrak{d}_1$	a_1	b_1	c_1	a_2	b_2	c_2	a_3	b_3	c_3	$\mathfrak{d}_4$	

Kleinste gemeinschaftliche Vielfache.

Definitionen:

$\mathfrak{d}''' = a+b+c$; $\mathfrak{d}_4 = a-b-c$

$a''' = b+c$, $b''' = c+a$, $c''' = a+b$; $a_3 = b-c$, $b_3 = c-a$, $c_3 = a-b$

$\mathfrak{d}' = a'''-b'''-c'''$; $\mathfrak{d}_1 = a_3+b_3+c_3$

$a'' = a+\mathfrak{d}'$, $b'' = b+\mathfrak{d}'$, $c'' = c+\mathfrak{d}'$; $a_2 = a-\mathfrak{d}_1$, $b_2 = b-\mathfrak{d}_1$, $c_2 = c-\mathfrak{d}_1$

$a' = a+\mathfrak{d}_1$, $b' = b+\mathfrak{d}_1$, $c' = c+\mathfrak{d}_1$; $a_1 = a-\mathfrak{d}'$, $b_1 = b-\mathfrak{d}'$, $c_1 = c-\mathfrak{d}'$

$a_0 = a'-\mathfrak{d}' = a_1+\mathfrak{d}_1$, $b_0 = b'-\mathfrak{d}' = b_1+\mathfrak{d}_1$, $c_0 = c'-\mathfrak{d}' = c_1+\mathfrak{d}_1$

Oder auch:

$a'' = b'''-c'''$, $b'' = c'''-a'''$, $c'' = a'''-b'''$; $a_2 = b_3+c_3$, $b_2 = c_3+a_3$, $c_2 = a_3+b_3$

$a' = a+a_2$, $b' = b+b_2$, $c' = c+c_2$; $a_1 = a-a'''$, $b_1 = b-b'''$, $c_1 = c-c'''$

$a_0 = a_1+a_2 = a'-a'''$, $b_0 = b_1+b_2 = b'-b'''$, $c_0 = c_1+c_2 = c'-c'''$

Roth unterstrichen sind die 2 · 117 Resultanten, welche von jeder [illegible] die übrigen 2 · 261 Stellen geben Theilbarkeiten. $261 + 117 = 378 = \frac{1}{2}\, 28 \cdot 27$

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Einfache Schritte und Classen-Anzahlen:

$$(d''', a''') = (b'', c'') = (c'', b'') = (a', d') = (a', a_0) = (a, a_1) = a$$
$$(d''', b''') = (c'', a'') = (a'', c'') = (b', d') = (b', b_0) = (b, b_1) = b$$
$$(d''', c''') = (a'', b'') = (b'', a'') = (c', d') = (c', c_0) = (c, c_1) = c$$
$$(a_3, d_4) = (c_2, b_3) = (b_2, c_3) = (d_1, a_2) = (a_0, a_1) = (a', a) = a_1$$
$$(b_3, d_4) = (a_2, c_3) = (c_2, a_3) = (d_1, b_2) = (b_0, b_1) = (b', b) = b_1$$
$$(c_3, d_4) = (b_2, a_3) = (a_2, b_3) = (d_1, c_2) = (c_0, c_1) = (c', c) = c_1$$

$$\left.\begin{array}{l}(a'', a') = (b'', b') = (c'', c') = (d', a_0) = (d', b_0) = (d', c_0) = \\ (a_1, a_2) = (b_1, b_2) = (c_1, c_2) = (a_0, d_1) = (b_0, d_1) = (c_0, d_1) =\end{array}\right\} = h \quad \underline{\text{oder } 2}$$

$$\begin{array}{l|l|l}(b, c) = hbc_1 & (c, b) = hcb_1 & (d''', d') = abc \\ (c, a) = hca_1 & (a, c) = hac_1 & (d', d_1) = h^2 \\ (a, b) = hab_1 & (b, a) = hba_1 & (d_1, d_4) = a_1b_1c_1\end{array}$$

$$(b, c)(c, a)(a, b) = (c, b)(a, c)(b, a) = h^2 abc a_1b_1c_1 = h(d''', d_4)$$

Den letzten Satz kann man auch ohne die vollständige Theorie der drei Moduln (Dualismus), [illegible] durch die drei Elementar-Sätze

I. $(a, b) = (a+b, b)$.

II. $(a, b) = (a, a-b)$.

III. Wenn y Divisor von u, und u Divisor von w, so ist $(y, w) = (y, u)(u, w)$

beweisen. In der That, mit Anwendung der obigen Bezeichnungen (nicht Sätze) ist

$(b, c) = (b+c, c) = (a'', c)$ zufolge I,

da ferner $a'' = b'''$ Multiplum von a''' und Divisor von c ist, so ist

$(a'', c) = (a''', c'')(c'', c)$ zufolge III,

ferner ist

$(a''', c'') = (a''', a''' - b''') = (a''', b''')$ zufolge II,

und aber ~~[illegible]~~ $a''' + b''' = (b+c)+(c+a) = b+c+a = a+b''$, mithin

$(a''', b''') = (a''' + b''', b''') = (b+b'', b''') = (b, b''')$ zufolge I,

und folglich ergiebt sich die erste der sechs Gleichungen

$$\text{IV.} \left\{\begin{array}{l}(b, c) = (b, b''')(c'', c);\ (c, a) = (c, c''')(a'', a);\ (a, b) = (a, a''')(b'', b) \\ (c, b) = (c, c''')(b'', b);\ (a, c) = (a, a''')(c'', c);\ (b, a) = (b, b''')(a'', a)\end{array}\right.$$

woraus die übrigen durch Permutation folgen; und hieraus ergiebt sich der obige Satz

V. $(b, c)(c, a)(a, b) = (c, b)(a, c)(b, a) = (a, a''')(b, b''')(c, c''')(a'', a)(b'', b)(c'', c)$

Ähnlicherweise ergiebt sich ebenso

$(b, c) = (b, b+c) = (b, a_3)$ zufolge II,

da ferner $b_2 = c_2 + a_3$ Divisor von a_3 und Multiplum von b ist, so ist

$(b, a_3) = (b, b_2)(b_2, a_3)$ zufolge III,

ferner ist

$(b_2, a_3) = (c_2 + a_3, a_3) = (c_2, a_3)$ zufolge I,

es ist aber $c_2 - a_3 = (a-b) - (b-c) = a-b-c = c-c_2$, mithin

$(c_2, a_3) = (c_2, c_2 - a_3) = (c_2, c - c_2) = (c_2, c)$ zufolge II,

woraus

$$\text{VI.} \left\{\begin{array}{l}(b, c) = (b, b_2)(c_2, c);\ (c, a) = (c, c_2)(a_2, a);\ (a, b) = (a, a_2)(b_2, b) \\ (c, b) = (c, c_2)(b_2, b);\ (a, c) = (a, a_2)(c_2, c);\ (b, a) = (b, b_2)(a_2, a)\end{array}\right.$$

und

VII. $(b, c)(c, a)(a, b) = (c, b)(a, c)(b, a) = (a, a_2)(b, b_2)(c, c_2)(a_2, a)(b_2, b)(c_2, c)$

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